MICROECONOMICS
Final exam – October 19^{th}, 2012
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3 hours – Documents and calculators NOT allowed

Exercise 1^{B}

Assume $L \geq 2$, let $\alpha, \gamma \in \mathbb{R}^L_+$ with $\alpha \gg 0$, and consider the consumption set

$$X = \{x \in \mathbb{R}^L_+ : x \geq \gamma\}$$

and a competitive consumer whose utility function $u : X \to \mathbb{R}$ is defined by

$$u(x) = \prod_{l=1}^{L} (x_l - \gamma_l)^{\alpha_l}.$$

(This is known as a Stone-Geary utility function.) Throughout this exercise we restrict attention to price-wealth situations (p, w) such that $p \gg 0$ and $w > p \cdot \gamma$. We can interpret γ as a "subsistence" bundle that has to be consumed in order to merely "survive", the consumer only drawing utility from the "residual bundle" $x - \gamma$.

1. To what kind of preferences over residual bundles does u correspond?
2. Why can you assume without loss of generality that $\sum_{l=1}^{L} \alpha_l = 1$? Do so from now on.
3. Show that the consumer's utility maximization program always has a unique solution and that this solution is always interior.
4. Determine the consumer's Walrasian demand function. Interpret. What is the shape of the wealth expansion path?
5. Determine the consumer's indirect utility function.
6. Without solving the consumer's expenditure minimization program, determine his expenditure and Hicksian demand functions.
7. Compute the own-price and cross-price partial derivatives of the Walrasian and Hicksian demand functions. Interpret their signs.
8. Verify that the Slutsky equation holds.

Exercise 2^{B}

Consider a competitive producer whose single-output, two-input technology is defined by the production function

$$q = f(z) = \max(z_l^{\frac{1}{4}}(z_k - 1)^{\frac{1}{4}}, 0),$$

where $z = (z_l, z_k) \geq 0$ denotes the vector of input quantities (l for labor and k for capital) and $q \geq 0$ denotes the output quantity. Let $w = (w_l, w_k) \gg 0$ denote the vector of input prices and $p > 0$ denote the output price.

1. What properties does the production set satisfy? How are the returns to scale?
2. Determine the conditional factor demand correspondences (or functions) $z_l(w, q)$ and $z_k(w, q)$ and the cost function $C(w, q)$.
3. Determine the marginal cost function $MC(w, q)$, the average cost function $AC(w, q)$, the efficient scale $\bar{q}(w)$, the minimum price $\bar{p}(w)$ enabling the producer to make a non-negative profit, and the supply correspondence (or function) $q(w, p)$. Represent them graphically.
4. Determine the profit function $\pi(w, p)$ and the (unconditional) factor demand correspondences (or functions) $\hat{z}_l(w, p)$ and $\hat{z}_k(w, p)$.
5. Restricting attention to price situations (w, p) for which the supply correspondence is single-valued and strictly positive, compute the effect of a marginal increase $dw_l > 0$ of the price of labor on the conditional and unconditional factor demands as a function of (w, p). Determine their respective signs and compare, for each input, the conditional and the unconditional effect. Interpret and illustrate graphically in the input space.

Exercise 3^{A}

Assume $L \geq 2$ and consider a competitive consumer whose Walrasian demand function $x(p, w)$ is defined by

$$x_l(p, w) = \frac{\sum_{k=1}^{L} \beta_{lk} p_k}{\sum_{k=1}^{L} p_k} \cdot \frac{w}{p_l} \text{ for } l = 1, \ldots, L,$$

where all β_{lk}'s are non-negative parameters.

1. For which values of the parameters is this demand function homogeneous of degree 0?
2. For which values of the parameters does this demand function satisfy Walras' law?

Exercise 4$^{\mathbf{A}}$

(This exercise is inspired by P N Sorrensen (2007), *Economic Theory* 31:367–370.) Consider a competitive consumer whose utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$ is defined by

$$u(x) = \min(x_1 + 10, 2(x_1 + x_2)).$$

Answer all the questions of this exercise graphically.

1. Draw (a representative subset of) the consumer's indifference curves.
2. Are the consumer's preferences monotone? strongly monotone? convex? continuous? homothetic?
3. Does the consumer's Walrasian demand satisfy the uncompensated law of demand? the compensated law of demand?

Exercise 5$^{\mathbf{A}}$

Consider a preference relation $\succsim$ on a set $X = \{x_n : n \in N\}$ of alternatives, where the index set N is a nonempty (finite or infinite) set of positive integers. The goal of this exercise is to show that $\succsim$ is complete and transitive if and only if there exists a utility function $u : X \to \mathbb{R}$ representing $\succsim$. We start by establishing the "only if" part.

1. Show that if there exists a utility function $u : X \to \mathbb{R}$ representing $\succsim$ then $\succsim$ is complete and transitive.

We now turn to the "if" part, so assume $\succsim$ is complete and transitive. For all $n \in N$, let $L(n) = \{m \in N : x_n \succsim x_m\}$ denote the (index set of the) lower contour set of x_n and let $|L(n)|$ denote the cardinality (i.e. the number of elements) of $L(n)$.

2. Show that if N is finite then the $u(x_n) = |L(n)|$ is a utility function representing $\succsim$.
3. Why is u no longer a utility function representing $\succsim$ if X is infinite?
4. (Harder) Find a utility function $\hat{u}$ that represents $\succsim$ whether N is finite or infinite.