Microeconomics – solutions to problem set 1

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Problem 1 *(Binary Relations, Preferences)*

a) A binary relation B on X is a set of ordered pairs $(x;y)$ with x, $y \in X$, or, equivalently, a subset of $X \times X$, $B \subseteq X \times X$.

Some properties of a binary relation:
Reflexivity: $(x;x) \in B$ for all $x \in X$
Irreflexivity: $(x;x) \notin B$ for any $x \in X$
Symmetry: $(x;y) \in B$ implies $(y;x) \in B$
Anti-symmetry: $(x;y) \in B$ and $(y;x) \in B$ imply $x = y$
Asymmetry: $(x;y) \in B$ implies $(y;x) \notin B$
Transitivity: $(x;y) \in B$ and $(y;z) \in B$ imply $(x;z) \in B$
Negative Transitivity: $(x;y) \notin B$ and $(y;z) \notin B$ imply $(x;z) \notin B$.

- $X = \mathbb{N}$, $(x;y) \in B$, if x and y have at least one digit in common:

Satisfies Reflexivity and Symmetry. Violates the rest of the properties.

- $X = \mathbb{R}^2$, $((x_1;x_2);(y_1;y_2)) \in B$, if $x_1 > y_1$, or if $x_1 = y_1$ and $x_2 > y_2$:

Satisfies Irreflexivity, Anti-Symmetry, Asymmetry, Transitivity, Negative Transitivity. Violates the rest of the properties.

- $X = \mathbb{R}$, xBy, if $|y - x| < 2$:

Satisfies Reflexivity and Symmetry. Violates all the other properties.

- $X = \mathbb{R}$, xBy, if $x + y$ is divisible by 3.

Satisfies only Symmetry and violates all other properties.

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b) xBy if and only if $u(x) > u(y)$. To see that asymmetry holds, assume xBy, and hence, $u(x) > u(y)$. It follows that $u(y) \not> u(x)$ and hence, $(y;x) \notin B$. To see that negative transitivity holds, note that $(x;y) \notin B$ and $(y;z) \notin B$ are equivalent to $u(x) \leq u(y)$ and $u(y) \leq u(z)$ and thus imply $u(x) \leq u(z)$, or $(x;z) \notin B$.

Problem 2 *(Choice Structures, WARP)*

Suppose that the choice structure $(\mathcal{B}; C(\cdot))$ satisfies WARP. Consider the following two revealed preference relations $\succ^*$ and $\succ^{**}$:

$y \succ^* x$, iff there is a $B \in \mathcal{B}$ s.t. $x, y \in B$, $y \in C(B)$, $x \notin C(B)$;

$y \succsim^* x$ iff there is a $B \in \mathcal{B}$ s.t. $x, y \in B$, $y \in C(B)$;

$y \succ^{**} x$ iff $y \succsim^* x$ and $x \not\succsim^* y$

a) To see that $\succ^*$ and $\succ^{**}$ represent the same preferences on X, let first $x \succ^* y$. Note that according to WARP, we have $y \not\succsim^* x$. Hence, by the definition of $\succsim^{**}$, $x \succ^{**} y$. Now assume that $x \succ^{**} y$. Then, there exists a $B \in \mathcal{B}$ such that $x \in C(B)$ and there is no $B \in \mathcal{B}$ such that $x \in B$ and $y \in C(B)$. Hence, $x \succ^* y$. Obviously, the first part of the argument uses WARP, hence the two preferences might differ if WARP is violated.

b) To see that $\succ^*$ need not be transitive, suppose that $\mathcal{B}$ consists of $B_1 = (x;y)$, $B_2 = (y;z)$ and $B_3 = (x;z)$ and let $C(B_1) = \{x\}$, $C(B_2) = \{y\}$ and $C(B_3) = \{z\}$. Obviously, this choice structure does not violate WARP, but the generated preference $\succsim^*$ violates transitivity.

c) Suppose that there are three elements, x, y and z such that $x \succ^* y$ and $y \succ^* z$. It follows that $x \in C(\{x;y\})$, $y \in C(\{y;z\})$. Consider $B = \{x;y;z\}$. If $x \notin C(\{x;y;z\})$, then by WARP, $y \notin C(\{x;y;z\})$ and hence, also by WARP, $z \notin C(\{x;y;z\})$. Since $C(\{x;y;z\}) \neq \varnothing$, it follows that $x \in C(\{x;y;z\})$. If $z \notin C(\{x;y;z\})$, then $x \succ^* z$ and transitivity is satisfied. Assume thus that $z \in C(\{x;y;z\})$, then $y \in C(\{x;y;z\})$. However, by WARP, this implies that for any set $\tilde{B}$ such that $y, z \in \tilde{B}$, $z \in C\left(\tilde{B}\right)$, in contradiction to $y \succ^* z$.

Problem 3 *(Choice Structures, Budget Constraint, WARP)*

a) The two conditions are:

if x_1 and x_2 are feasible at the year 1 income y_1 and the initial price vector p_1, then x_1 should not be feasible at $(y_2; p_2)$, where y_2 is the income in year 2;

if x_1 and x_2 are feasible at the year 2 income y_2 and the price vector p_2, then x_2 should not be feasible at $(y_1; p_1)$.

Consider the first case. Obviously, x_1 is feasible at $(y_1; p_1)$, and the budget constraint is satisfied with equality at x_1. Hence, for x_2 to be feasible at $(y_1; p_1)$, we need:

$$y_1 = p_1^s x_1^s + p_1^m x_1^m = 200 \geq p_1^s x_2^s + p_1^m x_2^m = 120 + 10x_2^m$$

or, $x_2^m \leq 8$. If this is the case, then x_1 should not be feasible at $(y_2; p_2)$, or

$$x_1^s p_2^s + x_1^m p_2^m = 180 > p_2^s x_2^s + p_2^m x_2^m = 120 + 8x_2^m$$

or $x_2^m < 7\frac{1}{2}$.

Now consider the second case. For both x_1 and x_2 to be feasible at $(y_2; p_2)$, we need that

$$p_2^s x_2^s + p_2^m x_2^m = 120 + 8x_2^m \geq x_1^s p_2^s + x_1^m p_2^m = 180$$

or $x_2^m \geq 7\frac{1}{2}$. Then, x_2 should not be feasible at $(y_1; p_1)$, or $x_2^m > 8$. We thus obtain that the admissible values for x_2^m are $x_2^m \in \left(0; 7\frac{1}{2}\right) \cup (8; +\infty)$.

b) If $x_2^m \in \left(0; 7\frac{1}{2}\right)$, then $x_1 \succ x_2$. If $x_2^m > 8$, then the reverse holds.

Problem 4 *(Stochastic Choice)*

List the possible preference orderings as follows:

1. $x \succ y \succ z$
2. $x \succ z \succ y$
3. $y \succ x \succ z$
4. $y \succ z \succ x$
5. $z \succ x \succ y$
6. $z \succ y \succ x$

and denote by $p_1 ... p_6$ the respective probabilities of these orderings. Then,

$$C(\{x; y\}) = C(\{y; z\}) = C(\{z; x\}) = (\alpha; (1 - \alpha))$$

implies

$$p_1 + p_2 + p_5 = p_1 + p_3 + p_4 = p_5 + p_4 + p_6 = \alpha$$

and

$$p_3 + p_4 + p_6 = p_2 + p_5 + p_6 = p_1 + p_2 + p_3 = 1 - \alpha$$

From these two conditions, we obtain

$$1 - 2\alpha = p_2 - p_4 = p_3 - p_5 = p_6 - p_1$$

or summing up,

$$p_2 + p_3 + p_6 = 3 - 6\alpha + p_1 + p_4 + p_5$$

Since $\sum_{i=1}^{6} p_i = 1$, and all $p_i \geq 0$, we have that $-1 \leq 3 - 6\alpha \leq 1$. Hence, the condition for the choice structure to be rationalizable is $\frac{2}{3} \geq \alpha \geq \frac{1}{3}$. Obvioiusly, this is satisfied for $\alpha = \frac{1}{2}$ (part a)), but not for $\alpha = \frac{1}{4}$ (part b)).