Microeconomics – problem set 1

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Problem 1 *(Binary Relations, Preferences)*

a) Define a binary relation. List some properties a binary relation can have. Check which of these properties are satisfied by the following binary relations:

- $X = \mathbb{N}$, $(x; y) \in B$, if x and y have at least one digit in common;
- $X = \mathbb{R}^2$, $((x_1; x_2); (y_1; y_2)) \in B$, if $x_1 > y_1$, or if $x_1 = y_1$ and $x_2 > y_2$;
- $X = \mathbb{R}$, xBy, if $|y - x| < 2$;
- $X = \mathbb{R}$, xBy, if $x + y$ is divisible by 3.

b) Assume that the binary relation B satisfies xBy if and only if $u(x) > u(y)$ for some real-valued function $u(\cdot)$. Show that B is a strict preference relation, i.e., it is asymmetric and negatively transitive.

Problem 2 *(Choice Structures, WARP)*

Suppose that the choice structure $(\mathcal{B}; C(\cdot))$ satisfies WARP. Consider the following two revealed preference relations $\succ^*$ and $\succ^{**}$:

$y \succ^* x$, iff there is a $B \in \mathcal{B}$ s.t. $x, y \in B$, $y \in C(B)$, $x \notin C(B)$;

$y \succsim^* x$ iff there is a $B \in \mathcal{B}$ s.t. $x, y \in B$, $y \in C(B)$;

$y \succ^{**} x$ iff $y \succsim^* x$ and $x \not\succsim^* y$

a) Show that $\succ^*$ and $\succ^{**}$ give the same preferences on X. Is this true if $(\mathcal{B}; C(\cdot))$ does not satisfy WARP?

b) Must $\succ^*$ be transitive?

c) Show that if $\mathcal{B}$ contains all subsets of X of up to three elements, then $\succ^*$ is transitive.

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Problem 3 *(Choice Structures, Budget Constraint, WARP)*

In two consequent years, 1 and 2, the prices of movie tickets, p^m and soccer game tickets, p^s are given by: $p_1^m = p_1^s = 10$, $p_2^m = 8$, $p_2^s = 10$. You observe that Bob has bought $x_1^m = 10$ movie tickets and $x_1^s = 10$ soccer game tickets in year 1 and $x_2^s = 12$ soccer game tickets in year 2. You do not observe Bob's entertainment budget in these two years, nor the number x_2^m of movie tickets he bought in the second year. You know however that in both years Bob spent his entire entertainment budget on soccer and movies and that Bob he satisfies the WARP axiom.

a) What constraints does this impose on the value of x_2^m?

b) For which values of x_2^m would you conclude that Bob prefers his consumption bundle from year 1 to his consumption bundle in year 2? For which values of x_2^m would the reverse be true?

Problem 4 *(Stochastic Choice)*

Solve problem 1.D.5 in MWG.