Microeconomics – problem set 2

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Problem 1 *(Binary Relations, Preferences, Indifference Curves)*

Consider the following binary relations / preferences defined on $\mathbb{R}^2_+$:

(i) $y \succ x$, if $y_1 + y_2 > x_1 + x_2$, or if $y_1 + y_2 = x_1 + x_2$ and $y_2 > x_2$;

(ii) $y \succ x$, if $y_1 > x_1$,or if $y_1 = x_1$ and $y_2 > x_2$;

(iii) $y \succ x$, if $|y_1 - y_2| > |x_1 - x_2|$;

(iv) $y \succ x$, if $y = (1; 1)$ and $x \neq (1; 1)$;

(v) $y \succ x$, if $x = (0; 0)$ and $y \neq (0; 0)$.

a) Illustrate each of these preference relations in a graph using indifference curves. Check whether the better-sets and the worse-sets are closed.

b) Define continuity of preferences. Which of the preference relations described above are continuous?

Problem 2 *(Preferences, quasi-concavity)*

Consider a preference relation $\succsim$ and a real-valued function u representing these preferences.

a) Suppose that $\succsim$ is convex. Show that u is quasi-concave.

b) Let u be quasi-concave. Check whether $\succsim$ is convex.

c) Consider the utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$:

$$u(x_1; x_2) = \max \left\{ \min \left\{ (x_1 + x_2); b \right\}; \frac{x_1 + x_2}{2} \right\}$$

(with $b > 0$). Illustrate $\succsim$ in a graph using indifference curves. Show that there is no concave utility function u representing these preferences.

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Problem 3 *(Preferences, monotonicity and non-satiation)*

Show that:

a) strict monotonicity implies monotonicity;

b) monotonicity implies non-satiation;

c) weak monotonicity, transitivity and non-satiation imply monotonicity. (weak monotonicty: $x \geq y \Rightarrow x \succsim y$.)

Problem 4 *(Homothetic and quasi-linear preferences)*

Show that

a) a continuous preference relation $\succsim$ defined on $X = \mathbb{R}_+^L$ is homothetic ($x \sim y \Leftrightarrow ax \sim ay$) iff it can be represented by a utility function which is homogeneous of degree 1 ($u(ax) = au(x)$);

b) a continuous preference relation $\succsim$ defined on $X = \mathbb{R}_+^L$ is quasi-linear w.r.t. the first commodity iff it can be represented by a utility function $u(x) = x_1 + v(x_2 ... x_L)$.