Microeconomics – problem set 3

Eric Danan*

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Problem 1 *(Utility Maximization, Kuhn-Tucker Method, Demand Theory, Quasi-Linear Utility)*

Chris consumes commodities $1...L$. His preferences for these commodities are described by a utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ given by:

$$u(x_1...x_L) = x_1 + \sum_{i=2}^{L} (x_i)^\beta$$

for some $\beta > 0$. Chris has an income of y and takes the price vector $p = (p_1...p_L)$ as given.

a) Write down Chris' utility maximization problem and illustrate it in a graph for $L = 2$. Use the Kuhn-Tucker method to solve for the Walras demand as a function of p and y (for $L = 2$).

b) Determine the income-offer curve of the household and draw it in a diagram.

c) Determine the indirect utility function of the household.

d) Assume now that $u : \mathbb{R} \times \mathbb{R}^{L-1}_+ \to \mathbb{R}$, i.e., good 1 can be consumed in both positive and negative quantities. Assume, as well that $p_1 = 1$. For an arbitrary L,

(i) show that the Walrasian demand for goods $2...L$ are independent of y;

(ii) determine how the demand for good 1 depends on y;

(iii) derive the indirect utility function and show that it is linear in y and separable in y and p.

Problem 2 *(Utility Maximization, Demand Theory, Constant Elasticity of Substitution)*

*THEMA, University of Cergy-Pontoise, CNRS. E-mail: eric.danan@u-cergy.fr. Webpage: www.ericdanan.net.

Barbara consumes only burgers (good 1) and chips (good 2). Her preferences for these two goods are given by the utility function:

$$u(x_1; x_2) = (\alpha_1 x_1^\rho + \alpha_2 x_2^\rho)^{\frac{1}{\rho}}$$

She has an income of y and takes the prices of burgers, p_1, and chips, p_2, as given.

a) Consider the following three cases: $\rho = 1$, $\rho \to 0$ and $\rho \to -\infty$. Specify Barbara's utility function for each of these three cases and draw the corresponding indifference curves. Provide an interpretation of the preferences represented by each of the utility functions.

b) Suppose that $\alpha_1 = \alpha_2 = \alpha$. Write down Barbara's utility maximization problem. Compute her Walrasian demand for burgers and chips, $f_1(p_1; p_2; y)$ and $f_2(p_1; p_2; y)$ and show that they are homogeneous of degree 0.

c) Derive Barbara's indirect utility function and show that it is:

- non-increasing in p_1 and p_2;

- increasing in y;

- homogenous of degree 0.

d) Compute Barbara's Walrasian demand as $\rho \to 0$. Derive her income-offer curve and plot it in a graph. Derive also the Engel curve for $p_1 = 1$ and plot it.

e) Barbara's elasticity of substitution between burgers and chips is given by:

$$\xi_{12}(p_1; p_2; y) = -\frac{\partial\left[\frac{f_1(p_1;p_2;y)}{f_2(p_1;p_2;y)}\right]}{\partial\left[\frac{p_1}{p_2}\right]} \frac{\frac{p_1}{p_2}}{\frac{f_1(p_1;p_2;y)}{f_2(p_1;p_2;y)}}$$

Show that Barbara's elasticity of substitution is constant in prices and income and only depends on ρ. Compute her elasticity of substitution for the three cases considered in part a).

Problem 3 *(Utility Maximization, Demand Theory, Perfect Complements)*

Anthony's preferences for goods $1...L$ are given by the utility function:

$$u(x_1...x_L) = \min\{x_1...x_L\}$$

He has an income of y and takes the prices $p_1...p_L$ as given.

a) Write down Anthony's utility maximization problem. For $L = 2$, illustrate it in a graph and solve it graphically.

b) For $L = 2$, show that the Barbara's Walrasian demand (from Problem 2) approaches Anthony's Walrasian demand as $\rho \to -\infty$.

c) Use your reasoning from part a) to derive Anthony's Walrasian demand for an arbitrary L.

d) Derive Anthony's income-offer curve.

Problem 4 *(Utility Maximization, Kuhn-Tucker Method, Demand Theory)*

Marianne's dinner consists exclusively of oysters (good 1), winkles (good 2) and whelks (good 3). The prices of these goods are p_1, p_2 and p_3 and Marianne's budget is y. Her preferences are given by the utility function:

$$u(x_1; x_2; x_3) = (x_1 - b_1)^{\alpha} (x_2 - b_2)^{\beta} (x_3 - b_3)^{\gamma}$$

a) Show that assuming $\alpha + \beta + \gamma = 1$ is without loss of generality. Formulate Marianne's utility maximization problem. What condition is necessary and sufficient to ensure that Marianne consumes strictly positive quantities of all three goods? Is it possible that Marianne spends less than y on her dinner?

b) Derive Marianne's Walrasian demand for oysters, winkles and whelks when the condition you derived in a) is satisfied. Show that it is homogeneous of degree 0 in prices and y.

c) Derive Marianne's indirect utility function.

Problem 5 *(Utility Maximization, Kuhn-Tucker Method, Demand Theory, Non-Satiation)*

With her afternoon tea, Alice likes cupcakes (good 1) and cookies (good 2). Her preferences are given by the following utility function

$$u(x_1, x_2) = 10 - (x_1 - 2)^2 - (x_2 - 2)^2$$

Alice can afford to spend y on pastries and the prices of cupcakes and cookies are given by p_1 and p_2.

a) Draw Alice's indifference curves in a graph. Check whether Alice's preferences satisfy monotonicity, strict monotonicity and local non-satiation. Formulate Alice's utility maximization problem. Use the Kuhn-Tucker method to derive conditions, under which:

(i) Alice consumes no cookies;

(ii) Alice consumes no cupcakes;

(iii) Alice spends less than y on pastries.

b) Derive Alice's Walrasian demand for cookies and cupcakes. Which of the standard properties of Walrasian demand correspondences does it satisfy and which ones does it violate?

c) Write Alice's indirect utility function. Which of the standard properties of indirect utility functions does it violate?

Problem 6 *(Utility Maximization, Demand Theory, Convexity)*

Peter is planning his next year vacation. He derives utility from hours spent surfing (commodity 1) and from hours spent skiing (commodity 2). His preferences for these two commodities are given by

$$u(x_1; x_2) = \alpha x_1^2 + x_2^2$$

He has allocated a budget of y to spend on his vacation. The price per hour of surfing is p_1, per hour of skiing — p_2.

a) Draw Peter's indifference curves in a graph. Do his preferences satisfy (strict) convexity? Formulate his utility maximization problem and illustrate it in your graph.

b) Derive Peter's Walrasian demand for skiing and surfing. Is his demand single-valued? Is it convex? Does Peter ever plan a vacation including both activities?

c) Derive Peter's income-offer curve, as well as his Engel curve for $p_1 = 1$.

d) Compute Peter's indirect utility function. Show that it is quasi-convex.