Microeconomics – problem set 4

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Problem 1 *(Expenditure Minimization, Kuhn-Tucker Method, Duality, Quasi-Linear Utility)*

Consider Chris from Problem 1 in Problem Set 3. Assume that Chris wants to obtain a utility level of at least $\bar{u}$.

a) Write down Chris' expenditure minimization problem and illsutrate it in a graph for $L = 2$. Use the Kuhn-Tucker method to solve for the Hicks' demand as a function of p and $\bar{u}$ (for $L = 2$).

b) Determine Chris' expenditure function.

c) Assume now that $\beta < 1$ and $u : \mathbb{R} \times \mathbb{R}_+^{L-1} \to \mathbb{R}$, i.e., good 1 van be consumed in both positive and negative quantities. Assume, as well that $p_1 = 1$. For an arbitrary L,

(i) show that the Hicks' demand for goods $2...L$ are independent of $\bar{u}$;

(ii) derive the expenditure function.

Problem 2 *(Demand and duality, Cobb-Douglas utility function)*

The utility function of a consumer is given by:

$$u(x_1; x_2) = x_1 x_2^2$$

where x_1 and x_2 denote the quantities of good 1 and 2. The prices of the goods are p_1 and p_2.

a) Derive the indirect utility function $v(p_1; p_2; y)$.

b) From the indirect utility function, derive the expenditure function $e(p_1; p_2; \bar{u})$.

c) State the cost minimization problem of the consumer for a given level of utility $\bar{u}$. Derive the Hicksian demand functions.

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d) Show that Shepard's lemma is satisfied, i.e. for $i \in \{1;2\}$, $h_i(p_1;p_2;\bar{u}) = \frac{\partial e(p_1;p_2;\bar{u})}{\partial p_i}$.

e) Show that for $i \in \{1;2\}$, $h_i(p_1;p_2;\bar{u})$ is homogenous of degree 0 in prices and decreasing in p_i.

f) Show that the expenditure function is:

*strictly increasing in p_1 and p_2; *strictly increasing in $\bar{u}$; *homogenous of degree 1 in prices.

Problem 3 *(Expenditure Minimization, Duality, Constant Elasticity of Substitution)*

Consider Barbara from Problem 2 in Problem Set 3. Assume $\alpha_1 = \alpha_2$ and $\rho < 1$. Barbara wants to obtain a utility level of at least $\bar{u}$.

a) State Barbara's expenditure minimization problem and derive Barbara's Hicks' demands for burgers and chips. Show that the Hicks' demands are homogeneous of degree 0 in prices and that the demand for good i is strictly decreasing in p_i.

b) Write down Barbara's expenditure function.

c) Since you have already solved Problem 2 in Problem Set 3, you have two alternative methods to get the Hicks' demands and the expenditure function. Show that these two methods give the same results.

Problem 4 *(Duality)*

As a selling representative, your statistics indicate that customer X has the following Walrasian demand functions for the L commodities you sell:

$$f_l(p;y) = \frac{\alpha_l y}{p_l}$$

with $\sum_{l=1}^{L} \alpha_l = 1$.

a) Derive the expenditure function of X.

b) Can you uncover X's utility function?

Problem 5 *(Duality)*

As a marketing director you are facing a mysterios customer (MC). All you know about MC is that their indirect utility function for the two products they consume is given by

$$v(p_1;p_2;y) = \frac{y}{p_1^{\frac{1}{3}} p_2^{\frac{2}{3}}}$$

a) Determine MC's Walrasian demand for these two products.

b) Derive MC's expenditure function.

c) Write down MC's Hicks' demands.

d) Can you determine MC's utility function?

Problem 6 *(Welfare Evaluation of Price Effects)*

In country A, the national railway company is offering a new customer service. A customer can purchase a railcard, which reduces the price per travelled kilometer from p_1^0 to p_1^1 on long-distance trains. For local trains, the price per kilometer remains unchanged at p_2. You are a customer of the national railways and have preferences over long-distance travel (x_1) and local travel (x_2) in km expressed by:

$$u(x_1; x_2) = x_1^{\alpha} x_2^{1-\alpha}$$

Your income (before purchasing a railcard) is given by y_0.

a) Your grandma gives you a railcard for your birthday. She wants to know how happy you are with the gift. Compute the change in your consumer surplus as a result of receiving the gift.

b) Without telling grandma, you decide to sell your newly obtained railcard on e-Bay. Your available income y_0 will increase to some y_1 (depending on the price you obtain), but you will not be able to use the price reduction on long-distance trains. What is the minimal sale price you would be willing to accept?

c) You don't have a railcard. What is the maximal price you would be willing to pay to obtain one?