Microeconomics – problem set 5

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Fall 2015

<u>Problem 1</u> *(Production Technology)*

a) Draw a production set that violates irreversibility and a production set that satisfies this property.

b) Suppose that Y is a production set associated with a single-output technology and $f(\cdot)$ is the corresponding (single-output) production function. Show that $f(\cdot)$ has constant returns to scale if and only if $y \in Y$ implies $\alpha y \in Y$ for all $\alpha \geq 0$, i.e., Y is a cone.

c) For a single-output technology, show that the production set Y is convex if and only if the corresponding production function $f(\cdot)$ is concave.

d) Consider a single-input, single-output technology and assume that the corresponding production function $f(\cdot)$ is differentiable and has increasing returns to scale. Show that f is convex and that the average product is increasing in input quantity.

<u>Problem 2</u> *(Profit Maximization, Cost Functions)*

Consider a firm which produces an output y using an input v. Its production technology is given by:

$$y \leq f(v) = \begin{cases} 4\sqrt{v-2}, & v > 2 \\ 0, & v \in [0; 2] \end{cases}$$

The price of the output is p and the price of the input is w. The firm takes these prices as given

a) Draw the firm's production set in a graph. Which of the standard properties of production sets does it satisfy?

b) Derive the firm's demand for the input and its supply of the output. Illustrate your analysis in a graph.

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c) Determine the cost function of the firm, as well as its marginal and average costs for all possible values of $w > 0$. Draw these functions in a graph.

d) Write down the profit function of the firm. Show that the profit function is convex and that the Hotelling's Lemma is satisfied for this production technology.

<u>Problem 3</u> *(Profit maximization with multiple factors of production)*

A firm produces a final good y using inputs v_1 and v_2. The firm uses the production technology
$$y \leq f(v_1; v_2) = 4v_1^{\frac{1}{4}} v_2^{\frac{1}{4}}$$
The prices of the inputs are w_1 and w_2, the price of the final good is p.

a) Determine the marginal product of each of the factors. What is the marginal rate of technical substitution? Does the production technology have increasing / decreasing or constant returns to scale?

b) Formulate the profit maximization problem of the firm and derive its factor demand. Derive the profit function of the firm.

<u>Problem 4</u> *(Profit maximization with multiple factors of production)*

A firm produces a final good y using inputs v_1 and v_2. The firm uses the production technology
$$y \leq f(v_1; v_2) = 2\sqrt{v_1} + v_2$$
The prices of the inputs are w_1 and w_2, the price of the final good is p.

a) Derive the marginal product of each input. Determine the marginal rate of technical substitution.

b) Formulate the profit maximization problem of the firm and derive its factor demand.

c) Derive the profit function of the firm.

<u>Problem 5</u>

Solve Problem 5.C.9 in MasColell, Whinston and Green.

<u>Problem 6</u> *(Profit Maximization, Factor Demand, Supply)*

A firm has a profit function given by:
$$\pi(p_1; p_2; w_1; w_2) = \frac{p_1^2}{4(w_1 - p_2)} + \frac{p_1^2}{4(w_2 - p_2)}$$
where $(p_1; p_2)$ is the output price vector and $(w_1; w_2)$ — the input price vector.

a) Show that the function π satisfies the properties of a profit function, i.e., it is non-decreasing in output prices and non-increasing in input prices and it is homogeneous of degree 1.

b) Derive the demand and supply functions of the firm.

c) What is the firm's production function?