Microeconomics – problem set 6

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Problem 1 *(Cost minimization, Conditional Factor Demand)*
A firm produces output y using inputs v_1 and v_2 and technology

$$y \leq f(v_1; v_2) = 2\sqrt{v_1 v_2}$$

The prices of the factors are w_1 and w_2, the price of the output is p.

a) Assume first that the input quantity of factor 1 is fixed at $\bar{v}_1 = 4$ and cannot be changed in the short-run. Derive the short-run cost minimization problem of the firm. Determine the demand for factor 2 and the short-run cost function.

b) Derive the short-run average and marginal cost functions and draw them in a $(c; y)$-diagram. Derive the short-run supply of y.

c) State the long-run cost-minimization problem of the firm. Derive the demand for inputs $v_1(w_1; w_2; y)$ and $v_2(w_1; w_2; y)$. Determine the cost function $c(w_1; w_2; y)$ and the optimal supply of y.

d) Show that there exists a unique output level $\hat{y}$ at which the short-run cost function is identical with the long-run cost function. Explain why the long-run cost can never exceed the short-run cost.

Problem 2 *(Cost Minimization, Conditional Factor Demand, Profit Maximization)*
A firm has a production function given by:

$$y(v_1 ... v_M) = \sum_{i=1}^{M} \sqrt{v_i}$$

where $y \geq 0$ is the only output of the firm and $v = (v_1 ... v_M) \in R_0^{M+}$ is the input vector. The input prices are given by $w = (w_1 ... w_M) \in R^{M+}$, the output price is $p > 0$. The firm takes market prices as given

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a) Write down the cost minimization problem of the firm. Illustrate it in a graph for the case $M = 2$. For which price vectors $(w_1...w_M; \bar{y})$ does the problem have a corner solution?

b) Determine the conditional factor demand function of the firm for arbitrary price vectors $(w_1...w_M)$. Check whether the conditional demand satisfies homogeneity of degree 0 in w.

c) Derive the cost function of the firm, its marginal and average costs. Show that the cost function is

(i) homogeneous of degree 1 and concave in w;

(ii) non-decreasing and convex in y.

d) Use the cost function to derive the conditional factor demand of the firm.

e) Determine the supply function of the firm and its profit function.

Problem 3 *(Cost Minimization)*

Solve Problem 5.C.10 in Mas-Colell, Whinston and Green.

Problem 4 *(Efficiency)*

In the year 101 996, Luna's space shuttle lands on the uninhabited planet Micron. Fortunately, her shuttle is equipped with a chocolate cookie machine, that produces chocolate cookies y using labor v (measured in light years) as input according to the following production function:

$$y \leq g(v) = \begin{cases} \frac{1}{16}v^2, & v \in [0; 12] \\ 9, & v > 12 \end{cases}$$

The rescue mission will only arrive after 12 light years and Luna's chocolate cookie reserves are exhausted. (Hence, her initial endowment consists of 12 light years time and 0 chocolate cookies.) Luna can use the time to either explore Micron or work at the chocolate cookie machine Luna is maximizing her utility from time devoted to exploration f and cookies k given by:

$$u(f; k) = fk$$

Consider the economy consisting of the utility maximizing Luna and the profit maximizing cookie machine. The price per chocolate cookie is p and the wage is normalized to $w = 1$. Luna owns the cookie machine and receives its profit.

a) Draw the production function in a graph. Does the function exhibit increasing / decreasing returns to scale? Is the production set convex?

b) State the profit maximization problem of the chocolate cookie machine and determine its demand for labor, its supply of cookies and its profit function. Illustrate the solution in a graph. Are there prices p, at which producing no cookies is optimal?

c) State Luna's utility maximization problem. Represent it in a graph. For a given profit of the cookie machine, derive her supply of labor and demand for cookies.

d) Is there a price p at which Luna's demand for cookies equals the supply? Which property of the production function is causing this result?

e) Find a production plan on the technology frontier, for which Luna's marginal rate of substitution equals the marginal rate of transformation of the cookie machine and illustrate it in your graph from part a). Does this production plan maximize Luna's utility subject to the production set of the economy? Is the production plan you found efficient? Is it profit-maximizing?

Problem 5 *(Production, Aggregation)*

Solve Problem 5.E.5 from Mas-Colell, Whinston and Green.